Kinetic scale turbulence and dissipation in the solar wind: key observational results and future outlook

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Turbulence is ubiquitous in the solar wind. Turbulence causes kinetic and magnetic energy to cascade to small scales where they are eventually dissipated, adding heat to the plasma. The details of how this occurs are not well understood. This article reviews the evidence for turbulent dissipation and examines various diagnostics for identifying solar wind regions where dissipation is occurring. We also discuss how future missions will further enhance our understanding of the importance of turbulence to solar wind dynamics.

1. Background

The solar wind is the outer atmosphere of the Sun, which is flowing outwards from the solar corona at supersonic and super-Alfvénic speeds, i.e. the solar wind speed, $V_{SW}$, exceeds both the speed of sound and the speed of the characteristic plasma mode, the Alfvén speed, given by $V_A = B/\sqrt{(4\pi\rho)}$, where $B$ is the magnitude of the local magnetic field and $\rho$ is the mass density. The equation is written in cgs units and $\rho$ includes all of the ions that comprise the solar wind. Typically, 80\% of the mass is carried by protons and virtually all of the rest is due to He
in the form of alpha particles. The alpha to proton mass density ratio is highly variable and so for any particular interval is likely to differ from the canonical value of 20% [1].

The existence of the solar wind was predicted in 1958 by Parker [2] and discovered at the dawn of the space age by Gringauz [3] in data obtained by the Luna spacecraft and by Neugebauer & Snyder [4] using data from Mariner 2. Early observations soon indicated that the solar wind contains magnetohydrodynamic (MHD) waves (in particular, Alfvén waves) [5,6] but in many other respects the fluctuations resemble fluid turbulence [7,8]. Coleman’s [8] paper inspired a lively debate as to whether or not the solar wind is best described as a medium consisting of non-interacting ‘waves’ or as a turbulent magnetofluid. One of the important observational aspects of the debate was the observation that the fluctuations in the magnetic and velocity fields are often nearly completely aligned so that \( \delta v = \mp \delta b / \sqrt{4\pi \rho} \), where \( \delta b \) and \( \delta v \) are the (vector) fluctuations of the magnetic and velocity fields, respectively. If one assumes that the fluctuations are totally aligned so that the equality is satisfied, then the ideal (i.e. zero dissipation) incompressible equations of MHD have no nonlinear terms and the equality is an exact solution (also see the discussion in [9]).1 In such a scenario, there cannot be any actively evolving turbulence. On the other hand, as pointed out by Coleman, the power spectra of the magnetic fluctuations resemble those seen in Navier–Stokes (incompressible) turbulence and the shape of those spectra is well described by Kolmogorov’s 1941 theory [10]. The cross helicity, defined as \( H_c = \frac{1}{2} \int d^3x \, \mathbf{v} \cdot \mathbf{b} \), quantifies this correlation.

(a) Early evidence for dissipation and turbulent heating

Further evidence that the solar wind is a turbulent magnetofluid was presented by Matthaeus & Goldstein [11], who pointed out that for (stationary) intervals spanning several days, the spectrum of magnetic fluctuations could be very close to the Kolmogorov \(-5/3\) power-law slope characteristic of turbulent fluids (e.g. Grant et al. [12]). A more intensive exploration of the ‘waves’ versus ‘turbulence’ question was carried out by Roberts & Goldstein [13,14], who concluded that both points of view had varying amounts of validity. In their analyses of intervals that spanned the heliosphere from 0.3 to 10 astronomical units (AU) using data from the two Helios spacecraft together with data from Voyager 1 and 2, they observed that for highly Alfvénic fluctuations [6] characterized by a high alignment between velocity and magnetic fluctuations (in the sense that the cross helicity was nearly maximal) there was little evidence of turbulent evolution, especially in fast solar wind out to 1 AU. (High cross helicity, along with strong correlations of other variables, tends to reduce the nonlinear terms in the MHD equations—see the discussion in [9].) However, in general, the degree of alignment tended to decrease with heliocentric distance. The evolution was more rapid for the short-scale fluctuations and slower for the larger-scale ones. Furthermore, the decline in cross helicity tended to be more rapid and larger for intervals that had been ‘stirred’ by co-rotating interaction regions, shock waves or other energetic phenomena. These results suggested that Alfvénic fluctuations were generated near the Sun followed by dynamical evolution caused by nonlinear couplings induced by the presence of large-scale structures as the wind flowed outwards into the heliosphere.

In a turbulent medium, energy injected at large scales must eventually dissipate at small scales. In a magnetofluid, the dissipation can occur in a number of ways. In collisional magnetofluids, dissipation occurs via viscous effects originating at the molecular level due to collisions between particles as well as by Joule heating (electrical resistivity). In collisionless plasmas, such as those encountered in near-Earth space, wave–particle interactions play a key role in mediating energy transfers between fields and particles. In the context of wave–particle interactions, one can imagine a scenario where the cascade of fluctuations to smaller and smaller scales reaches the point where the thermal plasma can be in resonance with the fluctuations and the resultant wave–particle processes can damp the electromagnetic energy, heating the plasma [15,16]. A related, but more ‘nonlinear’, picture might involve discontinuities, especially rotational, and/or current

1 Anti-correlation implies ‘waves’ propagating parallel to the background magnetic field while positive correlation implies propagation anti-parallel to the field.
sheets, where magnetic reconnection can occur. The interaction of the (demagnetized) particles with the small-scale fields would also result in plasma heating. As we shall point out below, differentiating between those two scenarios with the present instrumentation on spacecraft is not an easy task.

The dissipation of the turbulence due to velocity shears, wave damping, etc., will heat the plasma (see Coleman [8] for an early quantitative estimate of this process that was further developed by Hollweg [17]). Tu & Chen [18] used Helios data to investigate the radial evolution and concluded that turbulent dissipation fitted the observations quite well, unlike viscous dissipation, which failed to account for the observations by eight orders of magnitude.

The spherical expansion of the solar wind implies that it will cool adiabatically. That cooling competes with heating due to turbulent dissipation. Adiabatic cooling leads to a temperature decrease with radius $R$ that goes as $R^{-4/3}$. Gazis et al. [19,20] compared observations made by Voyager 1 as it travelled from 1 AU to nearly 10 AU with data from a very similar instrument on IMP 8 in Earth orbit. They showed that the decrease in plasma temperature was much slower than that predicted by the adiabatic law, decreasing only as $R^{-2/3}$. Gazis et al. surmised that the heating might be a consequence of thermal processes such as the addition of a heat flux due to heat conduction.

Verma et al. [21] investigated the possibility that the observed heating was due to the dissipation of the in situ turbulence. They showed that turbulent dissipation could provide sufficient heat to account for the observed slower-than-adiabatic radial dependence of the temperature. Verma et al. further concluded that the estimated heating was more consistent with the expectations of a Kolmogorov-like turbulent fluid cascade rather than a Kraichnan–Iroshnikov-like magnetofluid cascade [22,23]. However, the heating in the model was not quite adequate to account fully for the observed temperature dependence and the authors concluded that other processes, such as heat flux, might be involved.² To add to the complexity of this problem, Tu [24] found no evidence for turbulent heating in the time period he studied.

In the latest effort to include turbulent heating in the context of a global model of the solar wind [25], electrons, protons and interstellar pick-up protons were all given separate energy equations and the electron heat flux was incorporated using the approximation first derived by Hollweg [26,27]. While that model shows that turbulence does heat the solar wind, the paucity of observations in the outer heliosphere, coupled with the fact that the Voyager plasma instrument cannot see the pick-up proton temperature component, makes it difficult to draw definitive conclusions. Furthermore, the trajectory of Voyager 1 takes it out of the ecliptic plane and the model shows that high-latitude regions are hotter than is the ecliptic, which tends to obscure the effects of heating by turbulence. Consequently, it is important to examine in detail in situ measurements of the dissipation range of the turbulence to ascertain the detailed physical processes that heat the wind.

2. Heating and dissipation of solar wind turbulence

(a) Heating

One way to estimate heating of the solar wind is to measure the rate of the turbulent energy cascade [28,29], which is provided by measurements of the third moment of the fluctuations. The 10 years of solar wind data from 1998 to 2007 from the Advanced Composition Explorer (ACE) at 1 AU provided MacBride et al. [28] the opportunity to use third-order-moment theory by providing good convergence and statistically significant datasets [30]. MacBride et al. concluded that the total turbulent energy injection and dissipation rates derived agree with the in situ heating of the solar wind that is inferred from the temperature gradient. (For a discussion of the accuracy of third-order moments, see Podesta et al. [31].) The consistently high values of the MHD cascade

²One should keep in mind, however, that, in the solar wind, the heat flux is primarily carried by electrons and their coupling to the proton fluid is very weak, so the physical mechanism by which heat flux might contribute to the bulk ion temperature of the solar wind is unclear.
rates found by Stawarz et al. [29] indicated that the turbulent cascade deposited more energy than required for the observed proton heating, suggesting that the excess energy could potentially go towards heating electrons—which would be consistent with the Landau damping scenario mentioned above [32] (also see Sahraoui et al. [33]). In related work, Coburn et al. [30] found that the solar wind during the last solar minimum was still very turbulent, although the cascade rate of the turbulence was lower, perhaps owing to the paucity of high-latitude (coronal hole) sources.

In situations where the cross helicity of the fluctuations is large, one expects that the cascade rate should be slower than when the cross helicity is small because maximal cross helicity reduces all nonlinear interactions in the (incompressible, ideal) MHD equations (as mentioned above and as discussed in Matthaeus et al. [9]. Stawarz et al. [34] examined that expectation (also see [35]). They found that time periods containing large-helicity states indicated a significant back-transfer of energy from small to large scales. This back-transfer reinforced the dominance of the outward-propagating fluctuations. They suggested that the back-transfer might provide a partial explanation for large-helicity states in the solar wind, but acknowledged that such a back-transfer could not continue for long before it would come into conflict with the observed temperature of the solar wind at 1 AU. The mechanism for such a process is unclear, as in three-dimensional MHD turbulence energy cascades forwards and there is no observational evidence for any dynamical alignment of the velocity and magnetic fluctuations [13,14,36]. Furthermore, their conclusion has been criticized by Podesta [37], who suggested that Stawarz et al. may have underestimated the statistical errors inherent in their use of third-order moments.

Osman et al. [38] compared solar wind ACE data with results from MHD simulations and showed that regions characterized by high magnetic energy fluctuations (as detected by the partial variance of increments, or PVI) associated with the presence of coherent structures exhibit a higher mean ion temperature as compared with the ambient solar wind, suggesting that such structures can be potential sources of local plasma heating. (The reader is referred to [9, §5] for a detailed introduction to the PVI method.) By contrast, Borovsky & Denton [39], selecting hundreds of current sheets in the ACE magnetic field data, found no evidence of an enhancement in proton and electron temperatures during current sheet crossings. Further investigations are needed to resolve that apparent conflict.

The analyses that rely on the third-order law implicitly require that the turbulence be incompressible. However, the solar wind is not truly incompressible and, thus, errors in the observations of third-order moments, owing to their slow convergence to a mean, severely limit their use in the solar wind [31]. Coburn et al. [30] and Stawarz et al. [29,34] assumed that the computed third-order moments were accurate everywhere, and that the slow convergence was due to the highly variable nature of the third moment rather than to any large uncertainty in the observations. With this proviso, their work stands; however, this assumption has not been proven, and so there remains an unknown uncertainty on these results, making them inconclusive at the current time.

(b) Dissipation

Following the early and general investigations of the possible role of turbulence and other processes in heating the solar wind, interest turned to describing the dissipation range itself. The largest scales where dissipation is suspected to be happening are comparable to the gyroradius of a thermal proton, \( \rho_p = V_{th,p} / \Omega_p \) (where \( V_{th,p} \) is the proton thermal speed and \( \Omega_p \) is the proton gyrofrequency), or to the proton inertial length, \( \lambda_p = c / \omega_{pp} \) (where \( \omega_{pp} \) is the proton plasma frequency). These scales are typically between 100 and 1000 km in the solar wind at 1 AU and so are convected past a spacecraft in a couple of seconds or less at typical solar wind speeds. Thus, observations able to resolve the dissipation range must be made at cadences of the order of or faster than 1 Hz, which requires high-time-resolution magnetometer and plasma data.

In one of the pioneering studies of the dissipation range magnetic field, Leamon et al. [40] noted that the approach to the dissipation range at 1 AU sets in at spacecraft-frame frequencies of a few tenths of a hertz, a range accessible to magnetometers on Wind, Voyager and many other
Figure 1. An interplanetary power spectrum from the Wind spacecraft on 30 January 1995, 1300–1400 UT, showing the inertial range and the beginning of the dissipation range. (a) Trace of the spectral matrix with a break at ~0.4 Hz where the dissipation range sets in. (b) The corresponding magnetic helicity spectrum [41] (cf. [40, fig. 1]). The (Doppler-shifted) physical frequencies are indicated as coloured vertical lines on the frequency scale. Here \( f(\text{res}) = V_{SW} \Omega_{ci} / (V_A + V_{th}) \) (purple) is the parallel to \( B \) cyclotron wave resonance as described in Bruno & Trenchi [42], where \( \Omega_{ci} \) is the proton cyclotron frequency, \( V_A \) is the Alfvén speed and \( V_{th} \) is the thermal speed parallel to \( B \); \( f(d_p) = V_{SW} \Omega_{pi} / c \) (blue) is the proton inertial length, where \( \Omega_{pi} \) is the proton plasma frequency; \( f(\rho_p) = V_{SW} \Omega_{ci} / V_{th} \) (red) is the proton gyroradius; and \( f(c_p) = \Omega_{ci}(1 + V_{SW}/V_A) \) (green) is the Doppler-shifted proton cyclotron frequency.

The dissipation range should be characterized by a steepening of the power spectrum and, often, a change in the polarization or magnetic helicity spectrum of the magnetic fluctuations [43]. Whether or not the break seen at these proton scales actually reflects dissipation is difficult to determine despite the fact that, at these scales and smaller, kinetic effects are expected to couple the fluctuations to the background plasma, removing magnetic energy and thus heating the background plasma. Cyclotron damping and/or Landau damping may well be important, but dispersion due to finite Larmor radius effects, as reflected in either Hall MHD or gyrokinetic treatments [45], also leads to spectral breaks at these same ion scales. (For related ideas on the relationship between plasma distribution functions and dissipation, the reader is referred to [46–48].) It is clear, however, that to determine the physics of what is actually happening will require more and higher-time-resolution data in addition to detailed analyses.

Analysis of Wind data by Leamon et al. [40] showed that interplanetary turbulence consisting solely of parallel-propagating waves, such as Alfvén waves, was inconsistent with the observations. Two possibilities emerged: the fraction of the MHD turbulence that was two-dimensional could be interacting with ambient plasma (although the scales being analysed may have been too small for the application of MHD theory). Alternatively, kinetic Alfvén waves (KAWs) propagating at large angles to the background magnetic field were also consistent with the observations. KAWs are often observed to be quasi-oblique and nearly stationary (and, hence, quasi-two-dimensional). [4]

[3] The magnetic helicity represents the handedness of the fluctuations; see e.g. Moffatt [44] and Matthaeus & Goldstein [11].

[4] We will return to these ideas below when we discuss higher-time-resolution studies using search coil magnetometer data from the STAFF experiment on Cluster—data that showed breaks and/or curvature in the spectrum at scales much smaller than those associated with ion-scale physics.
In a subsequent paper [15], the authors examined the relative importance of cyclotron damping of quasi-parallel Alfvénic fluctuations and kinetic non-resonant dissipation. They concluded that, for the intervals analysed, the two processes were roughly equal, thus accounting for the observations that cyclotron damping, as indicated by magnetic helicity analyses, was never complete (cf. [40,43]). Furthermore, a study of the general characteristics of damping of KAWs [32] indicated that Landau damping by solar wind thermal protons and electrons must be important in determining the extent to which turbulent dissipation could heat the solar wind (for a recent analysis of the possible role of both whistler waves and kinetic Alfvén waves in the dissipation regime, see [49]).

Several studies of ion cyclotron waves in the solar wind have recently been published using data from the STEREO spacecraft. In the first, He et al. [50] explored the possible existence of Alfvén cyclotron waves and their coexistence with obliquely propagating KAWs or whistler waves. In a related study, He et al. [51] looked into the question of whether the beginning of the dissipation range of the spectrum was dominated by Alfvén, cyclotron or fast mode whistler waves. They concluded that the observed polarization of the magnetic fluctuations indicated that oblique KAWs dominated the spectrum near the spectral break. The interaction between magnetic helicity and instabilities is unclear, as there are many observed intervals of solar wind flow in which the temperature anisotropy and plasma $\beta$ are $\sim 1$ and so are completely stable to the excitation of cyclotron waves. In another study, Jian et al. [52] have reported extensive occurrences of ion cyclotron waves in the solar wind—the implications of these observations to solar wind heating and other processes are at present unclear. For a discussion of the possible role of alpha particles on instabilities that affect the population of solar wind fluctuations, the reader is referred to Podesta & Gary [53]. Further evidence that near the dissipation range the fluctuations are primarily oblique to the local magnetic field was published recently by Klein et al. [54] using data from Ulysses. Klein et al. concluded that some 95% of the fluctuating power near the dissipation scales was contained in perpendicular Alfvénic fluctuations, i.e. kinetic Alfvén waves.

The question of the role of cyclotron damping was also addressed from a different point of view by Leamon et al. [55] (also see [56,57]), where, instead of kinetic processes including Landau damping of KAWs, the comparison was with dissipation arising from the (perpendicular) turbulent cascade at the ion inertial scale evolving into thin current sheets that were the sites of magnetic reconnection (also see Osman et al. [58], who investigated the connection between the energy cascade rate of the turbulence and kinetic effects driven by temperature anisotropies). In the corona, at least (and maybe the solar wind, too), the conclusion was that a significant fraction of dissipation resulted from a perpendicular cascade and small-scale reconnection. In the solar wind, to study this regime requires use of high-frequency search coil data, as discussed in the following sections.  

(c) Determination of the scale of the spectral break between magnetohydrodynamic and kinetic turbulence

Since the physical mechanism that dissipates magnetic energy in the solar wind is still unclear, recent studies have tried to shed light on this question by exploring more deeply the physical processes operating where the Kolmogorov-like spectrum breaks and a steeper spectrum appears [40,75] (cf. figure 1). In this range, kinetic effects must be taken into account, and relating the break position to a physical scale is helpful for clarifying the process responsible for the high-frequency turbulent cascade observed.

Different mechanisms have been invoked to explain the kinetic range of the turbulent cascade. One possibility is that the ‘high-frequency’ (in the spacecraft frame of reference) range reflects a cascade of KAWs propagating almost perpendicularly to the mean magnetic field [75–77]

\footnote{In this review, we are concentrating on the description of the kinetic scale processes that dissipate the turbulence and heat the ambient fluid. The closely related subject of how waves and turbulence might accelerate and heat the corona cannot be addressed here and the reader is referred to the review in this issue by Cranmer et al. [59] and to other work that concentrates on that topic (e.g. [16,57,60–74]).}
beginning at the proton gyroradius. Another scenario envisages the proton cyclotron damping of Alfvénic fluctuations at scales of the ion inertial length, so that the high-frequency range has been interpreted as a cascade of whistler waves [78]. Furthermore, Hall MHD turbulence [79,80] and the generation of current sheets perpendicular to the mean field [81] also involve a spectral break near the proton inertial length.

Recently, Markovskii et al. [82], analysing a large sample of ACE solar wind data, have shown that the high-frequency spectral break is well correlated not with a single typical plasma scale but probably with a combination of scales and with the amplitudes of the magnetic fluctuations at those scales. This evidence suggests that the high-frequency cascade could be controlled by several processes. Using 1 s resolution Ulysses data obtained at radial distances greater than 1 AU together with 0.5 s resolution MESSENGER data for radial distances less than 1 AU, Perri et al. [83] studied the radial evolution of the high-frequency spectral break $f_b$ and concluded that the break frequency did not show any significant radial variation, perhaps indicating that the spectral break reflected a coronal imprint.

On the other hand, Bruno & Trenchi [42], using a combination of MESSENGER, Wind and Ulysses data during radial alignments of the satellites, concluded that there was a clear radial dependence, i.e. $f_b \sim R^{-1.09}$, and showed good agreement between the scaling of the observed break wavenumber and the scaling of the wavenumber corresponding to the resonance condition for parallel-propagating Alfvén waves [40], a result consistent neither with the damping of kinetic Alfvén waves [76] nor with Hall MHD effects [84,85].

In another look into the spectral break problem, Bourouaine et al. [86] used magnetometer data from Helios 2 and Ulysses to look at the radial variation of the magnetic power spectrum from 0.3 to 0.9 AU. The analysis focused on fast solar wind and the magnetic field spectral data extended up to 2 Hz. The Ulysses data indicated that, taking into account the two-dimensional nature of the turbulent fluctuations, the spatial scale corresponding to the frequency of the spectral break followed the proton inertial scale with distance. The observations indicated that the spectral break at the proton inertial scale might be related to the Hall effect or was controlled by ion cyclotron damping of obliquely propagating fluctuations or by the formation of current sheets (which could be responsible for ion heating through magnetic reconnection).

In a recent paper, Chen et al. [87] investigated the scale at which the first spectral break occurred in the plasma frame by dividing the intervals studied into groups in which the perpendicular ion plasma beta satisfied either $\beta_\perp \gg 1$ or $\beta_\perp \ll 1$. The conclusion was that for $\beta_\perp \ll 1$ the break occurred at $d_i$ while for $\beta_\perp \gg 1$ the break was at $\rho_i$. A recently published theoretical analysis [88] has pointed out the important role played by the magnetic helicity [41,44] in influencing the spectral index of magnetic power at both $d_i$ and $\rho_i$ scales.

Together, these analyses emphasize the need for new measurements, such as from Solar Orbiter and Solar Probe Plus, as well as from new missions to resolve this issue. Thus, at this time there is no definitive conclusion that can be drawn.

### 3. Determination of the three-dimensional properties of solar wind turbulence

A key measurement to make in turbulent magnetized plasmas is the determination of the gradients along and perpendicular to the mean magnetic field $\langle \mathbf{B}(t) \rangle = \mathbf{B}_0$. In Fourier space, this is equivalent to measuring the wavenumber spectra of the turbulence parallel and perpendicular to $\mathbf{B}_0$, i.e. $P(k_\parallel)$ and $P(k_\perp)$. This information is crucial for the direct identification of the plasma modes that carry the turbulence cascade. Such spectra also allow one to identify the nature of the wave–particle interactions that might be operating and that may be responsible for the local plasma heating. Unfortunately, measuring the full three-dimensional $k$-vector space of turbulence in the solar wind is challenging, especially since single-spacecraft data allow one to

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*One should keep in mind as discussed below that the Fourier transform of a series of discontinuities may also produce similar looking power spectra.*
measure only the component of the \( k \)-vector along the flow direction under the Taylor frozen-in flow assumption \cite{89}.

When data from only a single spacecraft are available, many approaches have been developed to measure the three-dimensional distribution of power in \( k \)-space. The techniques use either structure functions or wavelets to estimate the time- and scale-dependent amplitudes of fluctuations along with the direction of the ‘local’ magnetic field. The techniques require large deflections of the magnetic field to ensure sampling many angles relative to the local field direction. However, the techniques must also assume stationarity of the data. Most studies have focused on the inertial range of turbulence and have shown that the spectral scaling of fluctuations is anisotropic with respect to the local field direction, meaning that anisotropy of power grows as the turbulence cascades to smaller scales. As the cascade progresses, power in fluctuations perpendicular to the local magnetic field tends to dominate over power in the parallel direction \cite{90–94}. At least in a statistical sense, anisotropies generated in the inertial range will impact the dissipation and heating of the solar wind by determining the form of energy-containing fluctuations that reach scales where dissipation occurs. There is additional evidence that this anisotropy persists and even grows in the ion kinetic regime \cite{95}.

A three-dimensional measurement requires a minimum of four probes placed at the vertices of a fairly regular tetrahedron. The probes must simultaneously measure the fluctuating magnetic field. The ESA–NASA Cluster mission \cite{96} is the first space mission designed to measure three-dimensional structures in space, including the \( k \)-vectors of turbulence. These three-dimensional measurements have provided insight into the anisotropy of the turbulence and the actual mechanisms of energy dissipation. Analysis of the magnetic field data has employed the \( k \)-filtering (or wave telescope) technique \cite{97–101}, which is an interferometric method designed for multipoint measurements. It combines several time series recorded simultaneously at different points in space to estimate the four-dimensional function \( P(\omega, k) \) under the assumption that the time series is both stationary \cite{102} and spatially homogeneous.

An example of applying the \( k \)-filtering technique to Cluster data can be found in Sahraoui \textit{et al.} \cite{33}. The study indicated that the turbulence was strongly anisotropic at kinetic scales with \( k_\parallel/k_\perp \ll 1 \), and was dominated by KAWs with frequencies \( \omega/\omega_{ci} \ll 1 \), where \( \omega_{ci} = eB/m_\text{p}c \) is the proton gyrofrequency. Linear solutions of the Maxwell–Vlasov equations indicated that under those conditions energy dissipation was likely to happen through the Landau damping rather than via cyclotron resonance (see figure 2), primarily because the fluctuation wavevectors were too oblique to the magnetic field to resonate with solar wind protons. The very low frequency in the plasma frame of reference also implied that the ‘fluctuations’ were essentially stationary. This aspect of the turbulence was investigated further \cite{103} using the same dataset. That analysis showed the presence of current sheets with scales of the order of or smaller than the ion Larmor radius, suggesting that magnetic reconnection might be important within those current sheets in dissipating the energy. Similar fluctuations and current sheets have been found in Cluster data in both Earth’s magnetosheath \cite{104,105} and magnetotail regions \cite{106,107}. The coexistence of wave-like signatures and current sheets is an exciting topic of theoretical research that warrants further investigation.

The multi-spacecraft Cluster formation has also been used to investigate the three-dimensional structure of solar wind magnetic field fluctuations at small inertial range scales and in the proton kinetic regime. Narita \textit{et al.} \cite{108} used \( k \)-filtering to find the three-dimensional shape of the power distribution at small scales in the inertial range. Chen \textit{et al.} \cite{109} made a similar measurement using structure functions with data from a single spacecraft. Both results show highly anisotropic structures with a flattened pancake shape, elongated along the magnetic field direction with most power, or thinnest dimension, in the perpendicular direction aligned with the maximum variance direction in the mean-field-perpendicular plane. Chen \textit{et al.} \cite{110} also looked at the spatial anisotropy at ion kinetic scales using all four Cluster spacecraft. They concluded that the power and scaling of the anisotropy were similar to that predicted for KAWs.

An analysis of five years of Cluster (high time resolution) STAFF data \cite{111} found evidence that about 10% of the fluctuations show narrow-band, right-handed, circularly polarized fluctuations,
with wavevectors quasi-parallel to the mean magnetic field, superimposed on the spectrum of the permanent background turbulence. These observations provide evidence that when the electron parallel beta factor $\beta_e \parallel \geq 3$, the whistler waves are present at the heat flux threshold of the whistler heat flux instability. The presence of these whistlers indicates that the whistler heat flux instability may be contributing to the regulation of the solar wind heat flux, at least for $\beta_e \parallel \geq 3$, in slow wind, at 1 AU.

4. Observations of electron scale turbulence

Observations of AC magnetic fields from the search coil magnetometers on Cluster have been combined with the DC flux gate magnetometer measurements to provide broadband spectra from mHz to kHz in the spacecraft frame of reference (figure 3 [75,110,112,113]). The observations show a steepening of the power spectrum of the magnetic field between proton and electron scales (to approx. $-2.5$) with a further steepening of the spectrum (to approx. $-3.8$) at the highest frequencies sampled.

The high-time-resolution electric and magnetic field measurements on Cluster are available up to $\sim 180$ Hz and enable, for the first time, exploration of electron scale turbulence [75,114–116]. These observations show that magnetic energy has a spectrum that extends towards scales of order $\rho_e$, where dissipation becomes even more important as evidenced by the further dramatic steepening of the spectra ($\rho_e$ is the electron gyroradius, defined as $V_{th\,e}/\omega_{ce}$ where $V_{th\,e}$ is the thermal speed of the electron distribution and $\omega_{ce}$ is the electron gyrofrequency).

In addition to Cluster, the Spektr-R spacecraft is able to measure ion distribution functions with a cadence of 31 ms [117,118]. The Fourier transform of the velocity, density, and temperature time series are qualitatively similar to spectra produced from the magnetic field. All variables have a $\sim f^{-5/3}$ scaling in the inertial range, with a spectral break at a few tenths of a hertz to a more variable but steeper scaling between $f^{-2.5}$ and $f^{-3.5}$. In many cases [117,118], we observe a noticeable peak in the density and velocity spectra at the break scale, similar to a peak observed simultaneously in upstream (Wind MFI) magnetic field spectra.

Another approach to obtaining high-frequency ion density spectra in the solar wind is to measure the spacecraft potential. This has most recently been done using THEMIS [119], for which the electric potential can be measured at a cadence of 8192 samples per second. However, the conversion of the potential data into electron density is made complicated by the highly variable charging of the spacecraft by the solar wind. The resulting spectra are similar to the magnetic field spectra described above and are consistent with being dominated by KAW turbulence.

Figure 2. Observed dispersion relations (dots) at kinetic scales in the solar wind, with estimated error bars, compared with linear solutions of the Maxwell–Vlasov equations for three observed angles $\Theta$ (the dashed lines are the damping rates). The black curves are the different kinetic resonances (Landau and cyclotron effects). (Reproduced with permission from [33].)
Figure 3. (a) High cadence magnetic field spectra observed by combining data from the Cluster FGM and STAFF instruments. The different ranges of each instrument are highlighted showing how such a wide range of frequencies is covered. Proton and electron scales are marked and correspond to locations where the spectra steepen. Reproduced from [75, fig. 2]. (b) Power spectra of B measured by FGM and STAFF over five different intervals. The straight black lines are direct power-law fits of the spectra. Vertical arrows are the Doppler-shifted proton gyrofrequency and proton and electron gyroradii.

Additional studies of solar wind and magnetospheric turbulence have used alternative methods. Anisotropy has been investigated using wavelet techniques on data from single spacecraft [113] and structure functions from non-tetrahedral Cluster formations [110]. Those studies used the ‘local mean field’, that is, the mean magnetic field is measured over the envelope of the wavelet or structure function that is used to measure the fluctuations. This provides a scale-dependent mean magnetic field that allows many different angles to the mean field to be measured so long as a sufficiently long interval of stationary solar wind is processed. The results of these studies show that power anisotropy decreases (i.e. the fluctuations become less transverse, relative to the local mean field) [75,114], whereas the spectral index anisotropy increases in the dissipation range [110]. These analyses also support the idea that dissipation range fluctuations are dominated by KAWs.

Recent observations have motivated intensive research into electron scale turbulence both theoretically and numerically [120–127]. However, the underlying physics remains controversial primarily owing to the fact that the available observations are few and theoretical and numerical work has been extended to those small scales only recently. One issue concerns the scaling of the magnetic energy spectra down to and below $\rho_e$. Sahraoui et al. [75] first reported a power-law cascade $f^{-2.8}$ down to $f_{\rho_e}$, where a clear spectral break is observed, followed by another power law close to $f_{\rho_e}^{-4}$ ($f_{\rho_e}$ is the frequency in the spacecraft frame corresponding to the electron gyroscale when the Taylor frozen-in flow assumption is used). This has been confirmed in a survey of 10 years of STAFF waveform data in the free-streaming solar wind [128]. On the other hand, Alexandrova et al. [112,115], using STAFF-Spectrum Analyser (STAFF-SA) data, have reported exponential scaling in the scale range $k\rho_e \sim [0.03, 3]$.

Another controversy concerns the nature of the plasma modes that carry the turbulence cascade down to electron scales. Two main channels have been discussed: KAW [33,75,129,130] and the whistler turbulence [125]. There is growing evidence, as discussed above, that the dominant component of the turbulence is consistent with KAWs. This has been shown using the estimation of the ‘phase speed’ $E/B$ [75,129,131], the dispersion relation obtained from the $k$-filtering [33], the magnetic compressibility [113] and the density fluctuations [130]. Podesta [132] also concluded from several independent analyses that kinetic Alfvénic fluctuations are generally present in the solar wind. However, apart from the work using $k$-filtering [33], most other studies assume that the Taylor frozen-in flow hypothesis is satisfied. While this hypothesis is often valid
at MHD scales in the solar wind, it can fail for dispersive waves that have high phase speeds, such as parallel-propagating whistlers [133], and for intervals for which the fluctuation wavevectors are nearly orthogonal to the direction of solar wind flow.

The third point of controversy concerns the relative importance of the two paradigms used to describe turbulence at small scales: viz., the wave-like picture (KAWs, whistler waves, etc.) versus coherent structures (e.g. current sheets), and the possibility of the coexistence of both waves and structures in the turbulent mix. This question is closely related to the true nature of the turbulence cascade and to the appropriate theoretical approach for describing turbulence at kinetic scales—areas that will be addressed experimentally with the forthcoming launch of the Magnetospheric Multiscale (MMS) mission and are being intensively studied using numerical simulations and theoretical analyses.

5. Magnetic field discontinuities and intermittency

The solar wind is characterized by abrupt changes in the magnetic field, i.e. discontinuities, over a very broad range of time scales [103,134–136]. The properties of those time variations can be very different, so that according to their characteristics they have been interpreted as boundaries between flux tubes originating in the photosphere [137–139] or as current sheets that form as a consequence of the turbulent cascade [140] (also see [9]). The use of 450 vectors per second resolution STAFF data on Cluster allowed Perri et al. [103] to study the spatial properties of the magnetic field fluctuations, including current sheets, at scales close to and smaller than the proton inertial length. Those structures were seen as abrupt changes in the magnetic field direction and/or sharp decreases in the magnetic field magnitude (left panels of figure 4). The current sheet occurred at a time when the four Cluster spacecraft formed a regular tetrahedron with an average separation of 200 km.

Figure 4 displays an almost simultaneous observation of a current sheet by both Cluster 2 (red dashed line) and Cluster 4 (blue solid line) when the spacecraft separation was only 20 km along the flow direction. The angle of rotation \( \alpha(t, \tau) = \arccos(B(t) \cdot B(t + \tau))/(|B(t)||B(t + \tau)|) \), computed at a time scale \( \tau = 0.035 \text{s} \), increases in both Cluster 2 and Cluster 4. The magnetic field components are plotted in the local minimum variance reference frame (LMN) to better address signatures of the current sheet crossing (see figure legend). The L and N components lie in the current sheet plane along the maximum and the minimum variance directions, respectively. The magnetic field along L smoothly changes sign, while the component along N is almost zero. The intermediate variance M component is out of the plane and tends to show a bipolar signature, which is typical of the Hall field. The discontinuity has a spatial scale of \( d \sim 38 \text{ km} \) (very localized), which means for that interval \( \lambda_p \sim 1.6d \) and \( \rho_p \sim 1.9d \). A comparison with proton scale current sheets forming during a two-dimensional Hall MHD simulation of freely decaying turbulence gives qualitative agreement with solar wind observations (see right panels in figure 4), although discontinuities close to electron scales cannot be resolved. Notice that current sheets in the Hall MHD simulation are sites of magnetic reconnection. A challenging outlook is to find whether thin current sheets observed in the solar wind between proton and electron scales can be sites of magnetic energy dissipation via reconnection. This would be possible with high-resolution electric field data with a high signal-to-noise ratio. The present EFW data on Cluster allow for scientific analysis up to about 4–5 Hz, which is insufficient to resolve electron scale structures.

The presence of small-scale magnetic discontinuities seems to be a peculiar aspect of the magnetic field in the solar wind and has been detected both in the inner heliosphere and at 1 AU [141] by several methods, as the angle of rotation of \( B \) [142], the local intermittency measure (LIM) [143] and the PVI [144]. LIM and PVI permit identification of large-amplitude bursts of magnetic energy at different time scales and at given times, namely in regions where the concentration of magnetic energy exceeds the average value over the entire dataset.

On the other hand, the solar wind turbulent cascade at the MHD scales has been observed to be intermittent, that is, the distributions of the magnetic field increments \( \Delta B(t, \tau) = B(t + \tau) - B(t) \) become more and more non-Gaussian as the time scale decreases [145,146]. This implies that
Figure 4. (a) Magnetic field magnitude as measured by STAFF on board Cluster 2 (red dashed line) and Cluster 4 (blue solid line), (b) the $B_L$ component along the current sheet, (c) the $B_M$ component out-of-plane, (d) the $B_N$ component normal to the current sheet and (e) the angle of rotation $\alpha(t, \tau)$ as observed by all the four Cluster spacecraft. (f) Magnetic field components in the LMN reference frame from a two-dimensional Hall MHD simulation during a reconnecting current sheet, (g) the magnetic field intensity and (h) the angle of rotation. (Online version in colour.)

‘extreme’ values of the increments have a higher probability of occurrence as $\tau$ gets smaller. Those bursts in the magnetic field increments have been identified as regions of interface between portions of plasma having different speeds and bulk pressures, as MHD current sheets and as compressive discontinuities [138,147]. Thus, the turbulent cascade appears to generate localized structures that may extend to scales smaller than the proton inertial scale or gyroscale. The ion density data from Spektr-R also show pronounced intermittency [118,148], which increases, becoming very large in the inertial range, but stationary over the dissipation range. That behaviour matches that observed by the magnetic field [113,114].

The question now is how do magnetic structures observed at and below the proton scale influence intermittency at small scales? Recent studies on intermittency in that range of scales in the solar wind came to different conclusions: Alexandrova et al. [85] observed (using both FGM and STAFF data) a fourth-order moment of the distribution of the magnetic field increments that monotonically increased as the time scale decreased, indicating that the distributions became increasingly non-Gaussian. By contrast, Kiyani et al. [114], analysing a different solar wind interval of Cluster data, found that beyond the proton scale the distributions of the increments collapsed into a single shape, i.e. the statistics of the magnetic field increments becomes scale-invariant. A similar return to self-similarity has also been observed in both particle-in-cell (PIC) simulations and solar wind magnetic field data by Wu et al. [149]. Interestingly, a similar result was found by Chen et al. [148] using the data from Spektr-R [117]. Indeed, Chen et al. showed that the probability density functions of the plasma density increments are highly non-Gaussian at kinetic scales, but do not exhibit significant changes in the shape as a function of the time scale. Whether or not the origin of this time scale invariance in both the magnetic field and density in the kinetic range can be ascribed to sources of incoherent waves (cf. Wu et al. [149]) or to dissipation at current sheets (as suggested by Biskamp & Walter [150]) is not clear and needs further investigation.
6. Future missions that will address kinetic scale turbulence

As discussed above, high-resolution magnetic and plasma data obtained from Cluster and Spektr-R have allowed us to shed light on different new facets of kinetic turbulence, particularly as observed in the solar wind and Earth’s magnetosheath. However, the same studies have exposed the limitations inherent in the available data and have highlighted the need for improving both the field and particle instrumentation. This is imperative if we are to advance our knowledge of kinetic processes in magnetofluid turbulence in general and space and astrophysical plasmas in particular. Those limitations arise because of the very small amplitudes of both the electric and magnetic field fluctuations in the solar wind at the dissipation scale and the difficulty of measuring velocity distribution functions of low-energy protons and electrons with high time resolution. One of these limitations is illustrated in figures 5 and 6. Figure 5 shows statistical results obtained from the magnetic energy spectra in the free streaming solar wind acquired by the Cluster/STAFF search coil magnetometer (SCM) at 30 Hz ($k_{pe} \sim 0.5$). To obtain an accurate spectrum, a signal-to-noise ratio exceeding 10 is desirable. The histogram shown in figure 5 is constructed from a survey of 10 years of burst mode data. It is clear that at 30 Hz the signal-to-noise ratio is commonly less than 10. Similarly, figure 6 compares the sensitivities of several search coil magnetometers, including the STAFF instrument, with a magnetic spectrum taken by Cluster in the solar wind. The relatively low sensitivities of the MMS and THEMIS search coils compared with that of Cluster is related to mass/power constraints on those two missions (the lengths of the coils were 27 cm, 18 cm and 10 cm on Cluster, THEMIS and MMS, respectively). This example shows that the Cluster SCM remains the most sensitive magnetometer ever flown and allows one to come close to capturing very low-amplitude magnetic fluctuations in the solar wind (note, however, that at $\sim 100$ Hz, because the signal reaches the noise floor, one cannot measure the electron gyrofrequency $f_{ce} \sim 250$ Hz).

The next major advance in space instrumentation will come with the launch of the MMS mission in the spring of 2015. That mission is designed to study the physics of magnetic reconnection at physical scales, approaching the electron inertial length, depending on the local plasma parameters. The particle instrumentation will be the most advanced yet flown and will have outstanding temporal resolution (150 ms for ions and 30 ms for electrons). Achieving similar resolution on other missions will be challenging, in no small part because of the current limitations on telemetry rates. Furthermore, to detect low-energy electrons, one must use techniques such as the Active Spacecraft Potential Control as on Cluster and MMS.

The four MMS spacecraft will be able to manoeuvre much closer together than was possible with Cluster. However, the primary mission of MMS is confined to the magnetosphere and magnetosheath. By the time that the orbit of the four spacecraft precesses into the solar wind, the prime mission will have ended and possible support for an extended mission will not be known for a couple of years. It is likely, as can be seen in figure 6, that the intrinsic noise of the search coil magnetometer will not be as low as that on Cluster.

In addition to MMS, the DSCOVR mission, launched by NASA in February 2015, will have instrumentation capable of monitoring the solar wind plasma at 0.5 s resolution and the magnetic field at 0.02 s resolution. The plasma instruments will provide higher time resolution data, including distribution functions, than has been generally available, but will not reach down to the dissipation range. The forthcoming NASA Solar Probe Plus mission (scheduled for launch in 2018) and the ESA–NASA Solar Orbiter mission [151] (scheduled for launch in 2017) will both have magnetic and electric field instrumentation capable of exploring in detail the dissipation range of the turbulence in the inner heliosphere (on Solar Orbiter) and the solar corona (Solar Probe Plus). The Solar Probe Plus search coil and electric field instruments are expected to be at least as good as those on Cluster; the electric and magnetic field instrumentation on Solar Orbiter is similar to that of Solar Probe Plus. However, the particle measurements of these two missions will have relatively low time resolution, especially in comparison with MMS. Thus, as indicated from figure 6, there does not appear to be any approved mission that will have both fields and particle instrumentation adequate to solve fully the dissipation problem—more sensitive
Figure 5. The distribution of the signal-to-noise ratio (SNR) of the magnetic energy in the spectrum of $B_z$ (GSE) at $f = 30$ Hz. The data cover more than 10 years of Cluster STAFF burst mode data collected in the free streaming solar wind between 2001 and 2011. Note that SNR is defined as $\log(\delta B^2 / B_s^2)$ where $B_s$ is the sensitivity of the instrument at 30 Hz. Adapted from [128]. (Online version in colour.)

Figure 6. Comparison of sensitivities of search coil magnetometers on current and future missions. (The curve marked ‘TOR’ refers to an earlier proposal; the mission THOR now being studied will use that search coil magnetometer design.)

search coil magnetometers will be required for that task. A proposal designed to provide such an instrument is being prepared for a possible mission known as THOR. The proposal includes high cadence particle (especially electron) detectors. Measurements of three-dimensional (or even two-dimensional) electric field in the solar wind are indeed very challenging because of wake effects and the asymmetric photoelectron cloud. These two effects are reduced significantly for ‘Sun-pointing’ (i.e. the spin axis points towards the Sun) spacecraft, as proposed for THOR.

Acknowledgements. M.L.G. acknowledges the support of NASA headquarters to the Cluster mission.
Funding statement. R.T.W. is funded by a NASA GI grant and a NASA HSR grant at Goddard Space Flight Center. S.P.’s research is supported by ‘Borsa Postdoc POR Calabria FSE 2007/2013’. F.S. was supported, in part, by the project THESOW funded by L’Agence Nationale de la Recherche (ANR, France).

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