Coronal heating is at the origin of the EUV and X-ray emission and mass loss from the sun and many other stars. While different scenarios have been proposed to explain the heating of magnetically confined and open regions of the corona, they must all rely on the transfer, storage and dissipation of the abundant energy present in photospheric motions, which, coupled to magnetic fields, give rise to the complex phenomenology seen at the chromosphere and transition region (i.e. spicules, jets, ‘tornadoes’). Here we discuss models and numerical simulations which rely on magnetic fields and electric currents both for energy transfer and for storage in the corona. We will revisit the sources and frequency spectrum of kinetic and electromagnetic energies, the role of boundary conditions, and the routes to small scales required for effective dissipation. Because reconnection in current sheets has been, and still is, one of the most important processes for coronal heating, we will also discuss recent aspects concerning the triggering of reconnection instabilities and the transition to fast reconnection.

1. Introduction

The Solar Corona is a hot, tenuous fully ionized plasma which is kept at a temperature above $10^6$ K by a poorly understood mechanism which must non-radiatively transfer the abundant mechanical energy in the turbulent photospheric velocity field thousands of kilometers above, and then dissipate this energy within 1–2 solar radii.

Below the corona, the transition region and chromosphere is the site of extremely dynamic phenomena, and
as both spatial- and time-resolutions have increased, so has the extent of dynamical phenomena observed (Hansteen et al. [1]). The photosphere itself is the site of evolving turbulence, as displayed by its prominent multi-scale convection cell pattern dominated by granulation and super-granulation. Ultimately, the chromospheric/coronal heating and solar wind heating and acceleration problems are that of the transfer, storage and dissipation of the abundant energy present in motions in the photosphere. The key question is to identify how a small fraction of that energy is transported and stored, most probably via magnetic fields, to be released into particle and thermal energy above, sufficient to make up for losses into radiation and conduction by the corona in confined active region loops, with temperatures in excess of $2 \times 10^6$ K, and to provide the kinetic energy flux into the solar wind, for the darker, open coronal holes and other less apparent open regions whence the wind may stream.

Both emerging magnetic flux and the constant convective shaking and tangling of magnetic field lines already threading the corona contribute to these processes in what is an extremely structured, highly dynamic region of the solar atmosphere. In terms of energy flux, we can always think of a hypothetical spherical surface somewhere above temperature minimum and, calling $n$ the normal to this surface, the mechanical energy flux crossing it may be thought of as a Poynting flux

$$S \cdot n = \frac{c}{4\pi} (E \times B) \cdot n$$

$$= \frac{B^2}{4\pi} V_p \cdot n - \frac{B \cdot n}{4\pi} V_p \cdot B,$$

where the ideal Ohm’s Law has been used, plus a contribution from the energy directly associated with fluid motions which we have not written explicitly. The latter contribution, namely the advection of kinetic energy and enthalpy through acoustic type motions, is thought to contribute mostly via shock waves heating the chromosphere.

Now the energy in photospheric motions is dominantly at the large scales and low frequencies associated with granulation and super-granulation, with time-scales of minutes to hours and scales of hundreds to thousands of kilometres, which, when compared with the scales at which dissipation due to resistivity, viscosity or kinetic processes such as wave–particle interactions are involved, i.e. several hundred metres and frequencies in the kilohertz range, implies that there must be dynamical mechanisms acting to transfer the energy across scales: nonlinear interactions and a turbulent cascade are natural candidates. The subsequent evolution will depend on whether the magnetic field above is open or closed. For closed magnetic fields, there is a natural distinction between slow, quasi-static motions and wave-like, higher frequency motions, because how the corona responds to perturbations on a time-scale $\tau$ depends on the ratio $\tau/\tau_a$, with $\tau_a = l/V_a$ the time-scale for typical (Alfvén) wave propagation along a coronal loop. Roughly speaking, if $\tau/\tau_a \leq 1$ perturbations will propagate as waves, while if the inequality is reversed, the corona will respond quasi-statically. For the open corona, leading into the solar wind, all fluctuations will propagate upward as waves, but the gradients in density will lead to partial reflection, triggering a nonlinear cascade (we will not discuss the open corona here, but see, e.g. Verdini et al. [2] and Perez & Chandran [3], for a discussion of the role of turbulence and wave propagation in heating the corona and accelerating the wind).

The possibility of taking different approaches to explain the existence of small-scale structures in the corona has led to a somewhat artificial physical distinction between the corresponding mechanisms, classified as either wave or quasi-steady current dissipation theories. In reality, the dissipation of current sheets certainly involves the production of waves, and vice-versa, nonlinear evolution of waves in a structured medium can lead to the formation of current-sheets. Still, there is an interesting aspect relating to energy storage and release, concerning whether the dominant energy source is better modelled as a quasi-static process or a wave-like process: at lower frequency, energy is stored essentially in the magnetic field, and the kinetic energy within the coronal volume remains negligible. At higher frequency, the energy is more equally distributed between kinetic and magnetic energies of the fluctuations.
In the next section, we will summarize some of the work we have carried out on simulations of coronal heating within the framework of reduced magnetohydrodynamics (RMHD). This is not presented in the spirit of a review, but rather to illustrate both the progress and some of the important open questions that remain in understanding the coronal heating problem. For a review of numerical simulations, we encourage the reader to study the paper by Wilmot-Smith [4] in this same issue.

The accumulation of stressed magnetic field and the associated dynamics of current sheets naturally leads to the question of magnetic reconnection in the limit of large Lundquist numbers: in the third section, we will discuss the instability we have dubbed ‘ideal’ tearing, which naturally explains the transition to fast reconnection reconciling the nonlinear and linear dynamical points of view. Here the effects of viscosity and varying Prandtl number will also be described. Finally in the conclusions, we will briefly discuss and compare our results and scalings found to those found by other researchers using different models for the solar corona and its coupling to the lower atmospheric layers.

2. Magnetically dominated turbulence as a framework for coronal heating

Convective flows below the solar surface cause a random footpoint shuffling of magnetic field lines, which in regions of closed magnetic topology cause a secular increase in the stresses within the coronal magnetic field, as long as the timescale associated with the flows is longer than the propagation time along the loops in closed field regions. Parker [5] conjectured that such continuous footpoint displacement of coronal magnetic field lines must lead to the development of current-sheet discontinuities as the field continuously tries to relax to its equilibrium state, and that the dynamical interplay of energy accumulation via footpoint motion and the bursty dissipation in the forming current sheets would naturally result in a high temperature solar corona, heated by individual bursts of reconnection or nanoflares. The latter could be the extreme low-energy limit of all observed bursts of coronal energy releases. The number of flares as a function of total energy content, peak luminosity or duration all display well-defined power laws, extending over several orders of magnitude, down to instrumental resolution. Whether the observed statistics of events can account for coronal heating is still a matter of controversy, yet the robust power-law distributions are an extremely interesting indirect indicator of the general solar activity processes.

What then does turbulence have to do with the nanoflare heating scenario? Parker himself strongly criticized the use of the concept of turbulence, the formation of the current sheets being due in his opinion to the requirement for ultimate static balance of the Maxwell stresses. But the relaxation of the magnetic field toward a state containing current sheets must occur through local violations of the force-free condition, the induction of local flows and the collapse of the currents into ever thinner layers: a nonlinear process generating ever smaller scales. From the spectral point of view, a power law distribution of energy as a function of scale is expected, even though the kinetic energy is much smaller than the magnetic energy. The last two statements are clear indications that the word turbulence provides a correct description of the dynamical process.

Another important issue is whether the overall dissipated power tends to a finite value as the resistivity and viscosity of the coronal plasma become arbitrarily small. That this must be the case is at least plausible from simple physical considerations: suppose that for an arbitrary, continuous, footpoint displacement at the photosphere the coronal field were only to map this motion, and that there were no nonlinear interactions, i.e. the Lorentz force and convective derivatives were negligible everywhere. In this case, the magnetic field and the currents in the corona would grow linearly in time (e.g. [6]), until coronal dissipation would become strong enough, no matter how small the resistivity, at the scale of photospheric motions balancing the forcing. This implies that the amplitudes of the coronal fields and currents would be inversely proportional to resistivity, and the dissipated power, product of resistivity and square of the current, would also scale as the inverse power of the resistivity. In other words, the power dissipated in large-scale currents driven from the photosphere would become arbitrarily large! To emphasize this point, the smaller
the resistivity in the corona, the higher the power dissipated would be. But the amplitudes cannot become arbitrarily large, because nonlinear effects intervene to stop the increase in field amplitudes, increasing the effective dissipation at a given resistivity. Since the power cannot continue to increase monotonically as the resistivity is decreased, it is clear that at some point nonlinear interactions must limit the dissipated power to a finite value, regardless of the value of the resistivity: but finite dissipation at arbitrarily small values of dissipative coefficients is another definition of a turbulent system.

For most arbitrary photospheric footprint motions, the width of current sheets decreases exponentially in time (e.g. [7]), approaching 1 m within a few tens of Alfvén crossing times $\tau_A$. A typical value is $\tau_A = 40$ s, showing that this initial time is not only finite but also short compared with active region timescales of days to weeks. Once the steady state has been reached the current sheet collapse and current sheet reformation become ever coexisting processes. The nonlinear regime is therefore characterized by the presence of numerous current sheets, so that while some of them are being dissipated others are being formed, and a statistical steady state is maintained. It therefore seems that the Parker field-line tangling scenario of coronal heating may be described as a particular instance of magnetically dominated MHD turbulence.

In order to test this scenario, numerical simulations of increasing realism have been carried out. We defer a discussion of the ensemble of experiments carried out to the discussion, and focus here on the simplest possible numerical experiments that might be carried out: the simplest realization of this problem is to consider a coronal loop to be a finite volume of plasma, bounded by two conducting plates (photospheres) and threaded by a uniform magnetic field (a strong loop guide field), and then consider boundary motions at the plates that shear and attempt to tangle the initial field. This model lacks two important aspects of the real situation: (i) the curvature of the guide field, which leads to an outward net force tending to expand loops as energy in the transverse field increases; (ii) the possibility of storing energy directly in the distribution of the guide field, i.e. reconnection between loops of different sizes and or between closed and open field. In any case, many authors have developed coronal heating models with this preface based on the idea that an MHD turbulence cascade will result (e.g. [8,9] for theoretical models, and [10] for early numerical simulations). An even further simplification was used by Einaudi et al. [11] and Dmitruk & Gómez [12] in simulations considering just a two-dimensional cross-section of a loop, using a purely magnetic random forcing function to replace the effects of field line shuffling. In this way, they were able to show that a magnetically dominated turbulent cascade occurs in loops, and in addition proved that as a result of the dissipation occurring in a distribution of current sheets, the overall dissipated power was distributed in discrete events whose total energy, duration and peak intensity was indeed distributed according to power laws. Complementary work using shell models of turbulence, in which only the propagation along field lines is described in physical space, but the transverse domain is described using two-dimensional shell models of turbulence, were carried out by Nigro et al. [13] and Buchlin & Velli [14] confirming the power-law statistics of events and revealing the interesting phenomenon of ‘loop ringing’, whereby loop oscillations at their resonant frequencies followed burst of nano flares—a possible observable for the heating mechanism itself.

Numerical simulations of this process using the RMHD representation [15,16] have led to a more detailed understanding of the dynamics of field line tangling, an important new result being the derivation of scaling laws showing how the macroscopic heating rate depends on the dimension of closed loops and intensity of the magnetic field, and their connection to the resulting inertial range spectrum of magnetic fluctuations. The simulations showed how the MHD turbulence regime which develops may be thought of as a particular example of weak MHD turbulence, with magnetic energy dominating over the kinetic energy at the large scales (figure 1 shows RMS values of magnetic and kinetic energies for a typical simulation), and different spectra for kinetic and magnetic energy.

The magnetic spectra were found to be steeper than Kolmogorov, all the way to slopes of approximately 3, while the kinetic energy spectra were flatter. Such simulations at the same time yield a well-defined coronal oscillation spectrum associated with the turbulence. The spectrum is
not dominated by specific photospheric oscillation periods but rather has a power-law shape with superimposed overlapping resonances associated with the normal mode oscillations of the coronal loops. This dynamical scenario involves numerous random reconnection events embedded in the turbulence. This system was shown to exhibit heating rates that were on the lower limit, but compatible, with the heating rate required for closed coronal magnetic regions: considering photospheric velocity fields of magnitude 1 km s\(^{-1}\), convective eddies of size 1000 km and loop lengths of order 10 000–50 000 km, Rappazzo et al. [16] found the strongest heating rates to occur when the axial magnetic field was the strongest, and the turbulence weakest, with an RMS heating in such a case of about 1.6 \(\times\) 10\(^6\) erg cm\(^{-2}\) s\(^{-1}\). In the fully nonlinear stage at statistically steady state the Poynting flux, i.e. the energy that is entering the system for unitary time, balanced on time average the total dissipation rate. As a result, there is no average accumulation of energy in the box, beyond what has been accumulated during the linear stage, while a detailed examination of the dissipation time series (figure 1b) shows that the Poynting flux and total dissipations are correlated, with lag, in a statistical sense, but decorrelate around dissipation peaks. The fact that the heating rate tends to be at the lower level of what is required for active regions has lead some researchers to focus on similar models with forcing at the lower boundaries set at higher frequencies [17]. The alternative is that realistic physics might require a slightly more realistic model for the coronal turbulence, allowing for example for both footpoint motion with some net value of helicity, and therefore a further constraint on the turbulence (i.e. inclusion of the injection not only of energy, but of another invariant quantity such as magnetic helicity), or, alternatively, including the energy source of emerging flux. Be that as it may, the work of Rappazzo et al. was fundamental in discussing the types of turbulent regimes that might be set up in a coronal loop system.

For a long time, it was assumed that the current sheets forming in such simulations were essentially Sweet–Parker (SP, see the next paragraph) current sheets, being created and destroyed in about one nonlinear time. However, this created a problem when considering the limit of large Lundquist numbers: if SP current sheets can only dissipate an energy \(\propto S^{-1/2}\) in an Alfvén time, to obtain finite dissipation at infinite Reynolds number means that the number of sheets should grow as \(N \propto S^{1/2}\) or one would obtain zero dissipation in the ideal limit. However, runs with increasing \(S\) do not show such a growth, even though the number of \(x\)-points does increase dramatically with Lundquist number. Could there be something wrong with this interpretation of the observed state in the simulations?

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**Figure 1.** High-resolution simulation with \(V_a/u_{ph} = 200, 512 \times 512 \times 200\) grid points and \(Re = 800\). (a) Magnetic \((E_M)\) and kinetic \((E_K)\) energies as a function of time \((\tau_a = L/V_a\) is the axial Alfvén crossing time). (b) Poynting flux \((S)\) and ohmic \((J)\) and viscous \(\Omega\) dissipations. The inset shows a detail, with ohmic dissipation roughly correlated but lagging Poynting flux over time-scales of about 5 \(\tau_a\). Adapted from Rappazzo et al. [16].
3. Current sheets, ‘ideal’ tearing and fast reconnection

In older models of coronal heating, the dissipation of current sheets has often been discussed in terms of the SP steady-state reconnection scenario. However, as already realized by Parker [18] himself, the SP steady-state reconnection model was too slow to be able to account for any form of bursty energy release in the solar corona. Indeed, Petschek reconnection and its generalizations were developed to find a way around the impossible weakness of dissipation in high-temperature plasmas. In the same way, the resistive instabilities of current carrying equilibria were also found to be way too slow to explain the loss of confinement in toroidal devices, or explain the energy release required for flares and the analogous phenomena. However, with the increasing resolution of numerical simulations already Biskamp [19] had remarked on the possibility of fast reconnection within the framework of RMHD: he found that the SP stationary reconnecting current sheet became unstable to an extremely fast super-tearing, or plasmoid instability, once a critical value for the Lundquist number \( S = L V_a / \eta_m \sim 10^4 - 10^5 \) was exceeded (here \( L \) is the current sheet length, considered to be a macroscopic scale, \( V_a \) the Alfvén speed based on the equilibrium field far from the sheet and \( \eta_m \) is the magnetic diffusivity). Linear analysis confirmed the existence of this modified tearing instability, which was found to have a growth rate scaling with a positive exponent of the Lundquist number [20]. So from the impossibly slow reconnection rates of old, a new seemingly paradoxical fast reconnection scenario appeared on the horizon.

Pucci & Velli [21] argued for a solution to this paradox, and introduced the idea that as current sheets thin, the proper large Lundquist number limit, for a current sheet of macroscopic length, was one that would lead to an instability occurring on an ideal time-scale. In that paper it was shown that a current sheet with a limiting aspect ratio much smaller than that of the SP sheet, scaling as \( S^{1/3} \) (\( S \) being the Lundquist number), separates slowly growing resistively reconnecting sheets from those exhibiting fast plasmoid instabilities, and that this provides the proper convergence properties to ideal MHD, which is a singular limit of the resistive MHD equations. They obtained this result by studying the tearing mode instability for a family of current sheets with different aspect ratios. First of all, they pointed out that the renormalization of both the reconnecting mode growth rates (to the large scale time-scale \( L/V_a \) rather than the conventional tearing mode normalization to \( a/V_a \)) and the Lundquist numbers (to the large-scale \( L \) rather than the conventional sheet-length \( a \)) leads to a dependence of the growth rate on inverse aspect ratio \( \gamma \tau_a \sim (a/L)^{-3/2} \) (figure 2a). Then, they studied aspect ratios scaling as \( L/a = S^\alpha \) and found that as \( a \to \frac{1}{2} \) from below, the growth rate becomes independent of \( S \) itself, reaching a value of order unity (figure 2b). As such, that aspect ratio provides a physical upper limit to current sheet aspect ratios that may form naturally in plasmas. Otherwise, at large \( S \), the instability time-scale would become faster than the time required to set up the equilibrium in the first place. In other words, in the limit of large Lundquist numbers, SP current sheets should never form: as current sheets form and thin dynamically, a critical aspect ratio would be reached when they would break up again before reaching the SP scaling. Interestingly, if \( a < \frac{1}{2} \), the SP value, static equilibria may be constructed that do not diffuse, while flows in the SP model are required as the current sheet would otherwise diffuse on an ideal time-scale, based on the macroscopic current sheet length.

Again, the essential ingredient in the Pucci and Velli argument was the rescaling of the Lundquist number, which in the traditional tearing analysis is based on the thickness of the (macroscopic) current sheet equilibrium: the conjecture of Pucci and Velli was that their ‘Ideal’ tearing mode could not only explain the dependencies in simulations of the Parker nanoflare scenario, but also the trigger mechanism of more general explosive events such as flares. Indeed, the fast dependence of growth rate on the current-sheet aspect ratio, together with the large Lundquist numbers, would naturally lead to the on–off nature of magnetic reconnection and current sheet disruption that is strongly suggested by observations but was thought to be a problem because of the ‘slowness’ of the reconnection process and its scaling with Lundquists number in its traditional formulation.
Figure 2. (a) Growth rate $\gamma \tau_a$ as a function of the inverse aspect ratio for two different (large) values of the Lundquist number, showing a nonlinear scaling $\gamma \tau_a \sim (a/L)^{-3/2}$. (b) Growth rate as a function of the $a$-normalized $k$ for different Lundquist numbers and a current sheet whose inverse aspect ratio scales as $a/L \sim S^{-1/3}$. As the value of $S$ increases, individual dispersion relation curves shift to the left, i.e. have a maximum at lower $ka$, but the maximal growth rate reaches an asymptotic limit $\gamma \tau_a \simeq 0.623$ as shown by the dashed line. Adapted from Pucci & Velli [21]. (Online version in colour.)

Figure 3. Maximum growth rate versus aspect ratio for $S = 10^{12}$ at various Prandtl numbers. Dotted line represents the asymptotic growth rate at the critical inverse aspect ratio $a/L = S^{-1/3} P^{-1/6}$ while the dashed line corresponds to the maximum growth rate versus the inverse aspect ratio of the viscous SP sheet, $a/L_w \approx S^{-1/2} P^{1/4}$. Adapted from Tenerani et al. [22]. (Online version in colour.)

When comparing with nonlinear simulations, additional care however is necessary as usually a viscosity is included in the calculations: one might expect that introducing viscosity in the linear analysis might allow current sheets to contract further, returning to the possibility of reaching SP aspect ratios in a stable way. To extend the studies of ‘ideal’ tearing, Tenerani et al. [22] included the effects of viscosity, summarized here in figure 3. They showed how large values of the Prandtl number $P$ (ratio of Lundquist number to Reynolds number) could lead to the stabilization of large aspect current sheets, up to and including current sheets with SP aspect ratios. Their generalization of the Pucci–Velli critical aspect ratio for the ‘ideal’ tearing limit was shown to be the aspect ratio scaling as $L/a = S^{1/3} P^{1/6}$. The stabilizing effect of viscosity therefore allows for the formation of stronger magnetic shears in current sheets, and Tenerani et al. concluded that such effects might possibly lead to a smooth transition to kinetic regimes, once the critical width of sheets approaches the ion skin depth (or ion Larmor radius). While it has been realized for some time that at the coronal heating dissipative scale kinetic effects should be important, whether...
or not single `nanoflares', associated in the MHD turbulence scenario with the disruption of individual current sheets, required a fundamentally kinetic description at onset has been elusive. The results of Pucci–Velli and Tenerani et al. suggest the kinetic physics to be required not in the triggering of the disruption itself, most probably, but in the subsequent dynamics.

4. Conclusion

In this paper, we have described some aspects of the coronal heating problem, specifically in its relationship to magnetic reconnection and the MHD turbulence framework. We have discussed only simulations carried out within a well-defined set-up, namely the reduced MHD, two photosphere case, which has the great advantage of being essentially defined by a very small set of non-dimensional parameters. These are the ratio of the Alfvén crossing time to the photospheric forcing turnover time, the Lundquist number, and possibly, the frequency spectrum of the forcing. Considering the asymptotic extrapolation of the simulations to high Lundquist numbers, Rappazzo et al. [16] obtained scaling laws for coronal heating which have been used in large-scale simulations of the solar corona. These depend on the axial field strength $B^\beta$ where the exponent $\beta$ ranges from approximately 1.5 to 2.0.

Other works have focused on including other physical effects, either attempting to include convection, and simulating a three-dimensional cube including chromosphere, transition regions and corona, and therefore including also all the complexities of radiative transfer, heating and thermal conduction, gravitational stratification—with the advantage of being able to create synthetic images of the corona, see Bourdin et al. [23] and references therein, or by taking an intermediate step of remaining within the RMHD approximation but including stratification of a flux tube starting from the photosphere and rising into the corona [17]. We will not make general comments of the limitations of the full simulations (essentially resolution, so that it is difficult to discuss the energy spectrum seen and compare it to the one derived from RMHD), and the flux tube model of van Ballegooijen et al. [17]. The latter derive a linear scaling of the heating rate with $B$, but this depends on the spectrum of the fluctuations at the photosphere that they take, and the effects of the specific boundary motions and stratification. It will be important to understand the limitations in the latter case of RMHD and to carry out sufficient studies to reconcile the different results and their dependence on the typical parameters of the simulations. In particular, we have mentioned the importance of a more general description allowing for the dynamic of full MHD (i.e. including injection of helicity, emerging flux) in realistic geometries (curved loops and de-stabilization in such configurations).

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