Sound scattering by free surface piercing and fluid-loaded cylindrical shells

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A vertical flexible, thin, cylindrical shell is considered to be clamped to a rigid base in shallow water and piercing its surface. The shell is composed of an isotropic and homogeneous material and may be empty inside or filled with compressible fluid. Linear acoustics and structural dynamics are used to model sound scattering caused by an external incident sound wave. A solution is derived using a Fourier transform in the tangential and vertical directions. A collocation technique coupled with an orthogonalization procedure is used to account for the edge conditions of the shell. It is shown that zero sound scattering, indicating acoustic invisibility, is theoretically attainable and can be achieved when a continuous distribution of an oscillating pressure load is applied on the shell’s wall. Similarly, zero sound transmission into the shell’s inner fluid can also be considered. The possibility of using a pre-determined discrete distribution of the applied pressure load is also discussed. The derived equations are numerically solved to examine sound scattering by a thin aluminium shell in shallow water.

\textbf{Keywords:} general linear acoustics; structural acoustics and vibration; underwater sound

1. Introduction

This paper deals with the interaction between a flexible vertical cylindrical shell piercing a free surface and an externally generated sound field. Attention is directed towards the potential offered by the flexibility of the shell to reduce the scattering of sound caused by that interaction. The thin cylindrical shell is a structure of engineering interest across a vast variety of fields ranging from aeronautical to marine applications, such as modelling aircraft fuselages or underwater vessel hulls and offshore structures. Considerable research has been carried out on the interaction between fluid flow and a cylinder piercing the free surface (e.g. [1]). The interaction between an externally or internally generated sound field and a cylinder is also of importance owing to structural considerations, i.e. acoustic pressure loading, comfort for the crew inside the vehicle hull (modelled as a cylindrical shell) and detection purposes. The following paragraphs summarize some of the recent research on this subject.

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Sound scattering by an infinite rigid cylinder was initially investigated by Morse [2], who produced a Fourier series solution and elegant far-field asymptotic approximations. In the last few decades, interest has focused on the effects of the cylindrical shell’s flexibility and finite impedance owing to material coating on sound scattering and sound transmission through the shell’s walls. Flax & Neubauer [3] analysed analytically the scattering of a plane sound wave by an infinite elastic cylindrical shell coated with absorptive material. James [4] looked at the scattering of a plane wave propagating obliquely to a semi-infinite elastic cylindrical shell mounted on a rigid baffle and sealed by a rigid disc. A generalized Fourier series solution was used to find a combination of cylindrically and spherically spreading waves. Asymptotic analysis for long waves and heavy inner fluid loading was also given. Lee & Kim [5] considered an infinite cylindrical shell for sound transmission into the inner fluid by an impinging external plane sound wave. The shell was periodically stiffened, modelling the structure of an aircraft fuselage, and a generalized Fourier series solution was derived.

Mitri [6] studied analytically the acoustic radiation force caused by a plane sound wave impinging on an infinite cylindrical shell. The effects of a viscoelastic coating over the cylinder were considered. Brambley & Peake [7] looked at the stability of a cylindrical shell perturbed by the inner fluid and enforced by damping and spring forces from the outside. The latter configuration was used to model the structure of a jet engine intake layered with acoustic linings. A sudden spatial change from a rigid to a thin elastic shell wall was considered to analyse its effect on sound scattering. Caresta & Kessissoglou [8] studied analytically and numerically a finite cylindrical shell internally stiffened and sealed by a plate at each end, modelling a submarine hull. External forcing was considered to study the structural behaviour of the hull and the resulting radiated sound.

The reduction of sound transmission through a thin cylindrical shell into the inner fluid was studied by Thomas et al. [9]. A target function proportional to the acoustic energy within the shell was minimized using secondary force inputs. An example of a low-frequency sound transmission into the shell was studied to model aircraft propeller noise transmission into the fuselage and showed that a few secondary forces may be needed but this would be at the cost of an increase in the vibrational energy of the cylindrical shell. Constans et al. [10] suggested tailoring the shell structure by adding mass lumps in order to reduce sound radiation caused by the shell’s vibration. The use of pressure actuators mounted on the shell was considered by Maillard & Fuller [11] in order to reduce sound generated by the shell’s vibrations. A control loop for discrete structural acoustic sensing was suggested, generally achieving good global control over the frequency bandwidth of the shell’s first five structural modes. Li et al. [12] used generic algorithms to optimize the location of the pressure actuators in order to reduce interior low-frequency sounds caused by the shell’s vibrations. An internal floor partition was added inside the cylinder to better resemble the fuselage of an airliner.

The main objective of this study is the investigation of the potential of a shell’s flexibility to reduce sound scattering, often referred to as ‘making an object invisible to detection’. Traditionally, invisibility has been investigated in terms of radar detection, but recently interest has been extended towards other means such as optics and acoustics. Leonhardt [13] investigated analytically the material properties required to make a two-dimensional object invisible to light of wavelengths much shorter than the length scale of the object. Geometrical optics
and conformal mapping were used to find the required refraction index of the material and it was suggested that this method could also be applied to sound and other electromagnetic means of detection. Cummer et al. [14] considered analytically a three-dimensional acoustic shell, cloaking its interior from outside detection by preventing sound scattering in any direction and its transmission into the interior through the shell. The material specifications were derived, showing the need for an anisotropic mass density, which may be achieved using spring-loaded masses.

In our study, a thin shell is assumed to be composed of an isotropic and homogeneous metallic material, mounted on a rigid bed and piercing the shallow water surface. A schematic description of the problem is given in figure 1. The mathematical formulation of the problem and its solution are given next, followed by analysis of the results in §3 and some conclusions in §4.

2. Mathematical and numerical formulation

We consider both linear acoustics and linear shell dynamics. The thin circular shell is assumed to consist of an isotropic and homogeneous material. The shell may be filled with a compressible fluid and is taken as standing vertically, clamped to the floor bed and piercing the interface between the water and air, as illustrated in figure 1. The acoustic impedance of air is very low compared with the impedance of water, so that the air–water interface is considered to be a free surface, with zero pressure. The water is taken to be at rest or at least having such a velocity $U_0$ that is naturally much lower than the speed of sound and also having a Froude number $Fr_D = \frac{U_0}{U_0} \sqrt{\frac{gD}{}}$ that is much lower than 1, where $D$ is the cylinder’s diameter and $g$ is the acceleration owing to gravity. Thus, the deformation of the interface around the cylinder is sufficiently small to be approximated as a flat plane for acoustic considerations [15]. The floor bed is considered to have infinite acoustic impedance, a valid assumption for man-made or rocky non-porous surfaces. In reality, a seabed usually has finite impedance. This could potentially be incorporated in future studies by modifying the vertical distribution of the sound wave in the following analysis.

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Assuming a monochromatic incoming wave and taking the stationary wave equation as the governing acoustic equation, the incident sound wave can be taken as follows in the space–frequency domain:

\[ P_{\text{inc}}(r, \theta, z) = e^{ik_z r \cos \theta} \cos (k_{z,n} z), \quad k_{z,n} = \frac{(2n + 1)\pi}{2h}, \]  

where \( p_{\text{inc}}(r, \theta, z, t) = P_{\text{inc}}(r, \theta, z) \exp(-i\omega t) \). The vertical wavenumber \( k_{z,n} \) was taken as fulfilling the boundary condition of infinite impedance \( (\partial p/\partial z = 0) \) at the bed floor \( z = 0 \) and zero pressure at the free surface \( z = h \). The horizontal wavenumbers of the incoming wave satisfy

\[ k_x^2 + k_{z,n}^2 = \frac{\omega^2}{c_{\text{out}}^2}, \]  

where \( c_{\text{out}} \) is the speed of sound in the water outside the shell.

Adding the effect of the cylinder leads to the following solution for the acoustic pressure, \( P_{\text{ext}} \), outside the shell and in the space–frequency domain:

\[ P_{\text{ext}}(r, \theta, z) = e^{ik_z r \cos \theta} \cos (k_{z,n} z) + \sum_{m=0}^{M} \sum_{s=0}^{S} A_{ms} \cos (k_{z,m} z) \cos (s\theta) G_s(\gamma_m r), \]  

where \( P_{\text{ext}}(r, \theta, z, t) = P_{\text{ext}}(r, \theta, z) \exp(-i\omega t) \). The first term on the right-hand side of equation (2.3) represents the incident wave and the second term, expressed as a generalized Fourier series, corresponds to the scattered wave. In the following computations, the series will be truncated to a finite number of \( M \) and \( S \) terms.

The vertical wavenumber of the scattered sound wave \( k_{z,m} \) is related to its vertical index \( m \) in the same way that the vertical wavenumber of the incident sound wave \( k_{z,n} \) is related to the index \( n \).

The radial distribution of the scattered wave is

\[ G_s(\gamma_m r) = \begin{cases} H_s^{(1)}(\gamma_m r), & \omega/c_{\text{out}} > k_{z,m} \\ K_s(\gamma_m r), & \omega/c_{\text{out}} < k_{z,m}, \end{cases} \]

where

\[ \gamma_m = \sqrt{[(\omega/c_{\text{out}})^2 - k_{z,m}^2]}. \]  

\( H_s^{(1)} \) is the Hankel function of the first kind and \( K_s \) is the modified Bessel function of the second kind. Thus, when \( \omega/c_{\text{out}} > k_{z,m} \), the mode is radiative, i.e. decays algebraically in the far field, and when \( \omega/c_{\text{out}} < k_{z,m} \) the mode decays exponentially in the far field.

A solution for the transmitted sound wave into the compressible fluid inside the cylindrical shell can be similarly written as

\[ P_{\text{in}}(r, \theta, z) = \sum_{m=0}^{M} \sum_{s=0}^{S} B_{ms} \cos (k_{z,m} z) \cos (s\theta) \tilde{G}_s(\gamma_m r), \]  

where

\[ \tilde{G}_s(\gamma_m r) = \begin{cases} H_s^{(1)}(\gamma_m r), & \omega/c_{\text{out}} > k_{z,m} \\ K_s(\gamma_m r), & \omega/c_{\text{out}} < k_{z,m}, \end{cases} \]
and

\[
\tilde{G}_s(\gamma_m r) = \begin{cases} 
J_s(\gamma_m r), & \frac{\omega}{c_{in}} > k_{z,m} \\
I_s(\gamma_m r), & \frac{\omega}{c_{in}} < k_{z,m},
\end{cases} 
\tag{2.7}
\]

where \(J_s\) and \(I_s\) are Bessel functions of the first and second kind, respectively, and \(c_{in}\) is the speed of sound in the compressible fluid inside the shell. The scattered and transmitted wave coefficients \(A_{ms}\) and \(B_{ms}\) are determined by matching the normal acoustic velocity with the normal velocity of the shell and thus, by Junger & Feit [16], this leads to

\[
\frac{\partial P_{\text{ext}}}{\partial r} \bigg|_{r=a} = \rho_{\text{ext}} \omega^2 U, \quad \frac{\partial P_{\text{in}}}{\partial r} \bigg|_{r=a} = \rho_{\text{in}} \omega^2 U, \tag{2.8}
\]

where \(a\) is the cylinder’s radius, \(\rho_{\text{ext}}\) is the water’s density, \(\rho_{\text{in}}\) is the density of the compressible fluid inside the shell and \(U\) is the radial deflection of the cylindrical shell.

Following Junger & Feit [16], the governing equations for radial, tangential and vertical deflections \((U, V, W)\) of the thin shell are

\[
\left[ \frac{\rho_{\text{sh}}(1 - v^2)\omega^2}{E} - \frac{1}{a^2} \right] U - \frac{1}{a^2} \frac{\partial V}{\partial \theta} - \frac{v}{a} \frac{\partial W}{\partial z} - \frac{t^2 \nabla^4 U}{12} = -\frac{1 - v^2}{E} (f + q), \tag{2.9}
\]

\[
\frac{1}{a} \left( \frac{1}{a} \frac{\partial U}{\partial \theta} + \frac{1}{a} \frac{\partial V}{\partial \theta^2} + \frac{v}{a} \frac{\partial^2 W}{\partial z \partial \theta} \right) + \frac{(1 - v)}{2} \left( \frac{\partial^2 V}{\partial z^2} + \frac{1}{a} \frac{\partial^2 W}{\partial \theta^2} \right) + \frac{\rho_{\text{sh}}(1 - v^2)\omega^2}{E} V = 0
\tag{2.10}
\]

and

\[
\frac{\partial^2 W}{\partial z^2} + \frac{v}{a} \left( \frac{\partial W}{\partial z} + \frac{\partial^2 V}{\partial z \partial \theta} \right) + \frac{(1 - v)}{2a} \left( \frac{\partial^2 V}{\partial z \partial \theta} + \frac{1}{a} \frac{\partial^2 W}{\partial \theta^2} \right) + \frac{\rho_{\text{sh}}(1 - v^2)\omega^2}{E} W = 0, \tag{2.11}
\]

where \(E\) is Young’s modulus and \(v\) is the Poisson ratio, \(\rho_{\text{sh}}\) is the cylinder’s material density, \(f\) denotes any externally distributed load acting on the cylinder and \(q\) is the resultant acoustic pressure acting on the shell, i.e. \(P_{\text{int}} - P_{\text{ext}}\). The wall thickness \(t\) is assumed to be much smaller than the cylinder radius \(a\). No tangential traction is assumed at the shell’s surface. The equations for a cylindrical membrane are the same as equations (2.9)–(2.11) minus the term \(t^2 \nabla^4 U/12\) in equation (2.9). The cylinder is clamped to the floor bed \(z = 0\); thus,

\[
U = V = W = \frac{\partial U}{\partial z} = 0. \tag{2.12}
\]

The top of the cylinder \(z = h_{\text{cyl}}\) is considered to be free so that all stress resultants are zero, i.e. the vertical, shearing and membrane forces are zero as well as the bending moment perpendicular to the vertical direction [17].
The deflection of the shell can be expanded as a Fourier series in the tangential direction,

\[ U(\theta, z) = \sum_{s=0}^{S} U_s(z) \cos(s\theta), \quad V(\theta, z) = \sum_{s=0}^{S} V_s(z) \sin(s\theta) \] (2.13)

and

\[ W(\theta, z) = \sum_{s=0}^{S} W_s(z) \cos(s\theta). \] (2.14)

Similarly, the incident wave on the cylinder can be expanded as

\[ e^{ikr \cos \theta} \cos(k_{z,n}z) = \sum_{s=0}^{S} p_s(r) \cos(s\theta) \cos(k_{z,n}z), \] (2.15)

and the external pressure load \( f \) acting on the shell as

\[ f(\theta, z) = \sum_{s=0}^{S} f_s(z) \cos(s\theta). \] (2.16)

Substitution of the above series expansions into the equations for the shell deflection (equations (2.9)–(2.11)) yields a set of linear ordinary differential equations for \( U_s, V_s \) and \( W_s \), assuming that \( A_{ms} \) and \( B_{ms} \), the scattered and transmitted wave coefficients, are known. This set of linear equations can be solved analytically or numerically, as in this study, where central finite difference approximations of five point stencils are used. The result is a linear dependence between \( U_s \) and \( A_{ms}, B_{ms} \), \( f_s(z) \) and \( z \), found by solving the deflection equations (2.9)–(2.11).

Requiring equations (2.17) and (2.18) to hold at \( M \) grid points uniformly spaced along the cylinder’s vertical direction will yield a matrix equation with \( 2M \) unknowns, i.e. \( A_{ms} \) and \( B_{ms} \), assuming that the external load \( f_s(z) \) is known. This set of equations can be solved using the LU algorithm [18]. However, experience has shown that the matrix was often ill-posed, causing difficulties in using the solver. Orthogonalization of equations (2.17) and (2.18) with respect to \( \cos(k_{z,m}z) \) solved that problem.

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The above procedure yields a solution to the linear equations (2.17) and (2.18) for the deflection components of the shell, $U_s(z)$, $V_s(z)$, $W_s(z)$, and the scattered and transmitted wave coefficients, $A_{ms}$ and $B_{ms}$, respectively, assuming the incident sound wave and the applied pressure $f$ on the shell are known. To investigate the possibility of reducing or even cancelling sound scattering, the equations need to be rearranged by taking the incident and scattered waves as known and demanding that the scattered wave be equal to zero. The unknowns are the deflection of the shell, the transmitted sound wave and the externally applied pressure load $f_s(z)$. Similarly, one can seek a solution for a zero transmitted wave, i.e. $B_{ms} = 0$, by taking the unknowns as the scattered wave, the shell deflection and the applied pressure load $f$.

If limitations are imposed on the distribution of the applied pressure $f$, full cancellation of the scattered sound is no longer possible. Instead, the optimum distribution may be found by minimizing the target function $Q$

$$Q = \sum_{m=0}^{M_r} \sum_{s=0}^{S_r} A_{ms}^2,$$  

(2.19)

where $Q$ is proportional to the far-field acoustic energy by limiting the summation to the radiative part of the scattered wave. Thus, $M_r$ and $S_r$ denote the highest terms that are still radiative, i.e. do not decay exponentially in the radial direction. The optimization can be achieved via variation of $f$ while keeping whatever limitation is imposed, then calculating $A_{ms}$ using equations (2.17) and (2.18) to yield $Q$ and feeding it back into equation (2.19). In this study, Powell’s optimization algorithm was used since it is a set direction algorithm and is widely available [18].

3. Results and discussion

A thin circular cylindrical shell of diameter $a = 30$ cm and thickness $t = 0.5$ cm was considered. The shell was mounted in shallow water with depth $h = 8$ m. This case can represent an offshore pipe or the periscope of an underwater vessel. For illustration purposes, the shell was taken to be made of aluminium with a Young modulus of $E = 69$ GNm$^{-2}$, a Poisson ratio of $\nu = 0.3$ and density $\rho_{sh} = 2700$ kgm$^{-3}$. The water density was taken as 1000 kgm$^{-3}$ and the speed of sound as that in fresh water, i.e. $c = 1500$ ms$^{-1}$. The incident amplitude of the sound wave was taken as 1 Pa.

Contours of the pressure modulus of the sound field outside a rigid cylinder are shown in figure 2a for an incident sound wave of 5 kHz and a vertical wave index of $n = 0$ (see equation (2.1)). A V-shaped wake is clearly seen behind the cylinder, while reflected sound waves are present in front with some reflection to the side lines as well. Although the cylinder’s diameter is the same as the incident sound wavelength, this description of the sound field agrees qualitatively with Morse’s [2] description of a sound field scattered by a two-dimensional rigid highly non-compact cylinder, i.e. where the diameter is much larger than the wavelength. The sharp-edged shadow region was referred to as an interfering wave, while the scattered wave spreading in front of the cylinder was labelled as a reflected wave. We will use here the same terminology. In fact, adaptation of
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Figure 2. Contours of the pressure modulus $|P|$ plotted for (a) a rigid cylinder and frequency of 5 kHz, (b) an empty aluminium cylindrical shell of 0.5 cm thickness and frequency of 5 kHz, and (c) the same empty aluminium cylindrical shell and frequency of 10 kHz. The water’s depth $h$ and the cylinder’s height $h_{cyl}$ are 8 m. The cylindrical shell’s diameter is 0.3 m and the vertical index of the incoming sound wave is $n = 0$ (see equation (2.1)).

the two-dimensional solution of Morse [2] to the three-dimensional case of this study by modifying the horizontal incoming wavenumber according to equation (2.2) presents perfect agreement for the case of a rigid cylinder.

The effects of flexibility on an empty shell were first calculated by finding the influence matrix of the scattered and transmitted wave coefficients on the shell’s deflection, as outlined in §2. Artificial data were used to validate the solution. The results for the sound fields of the frequencies of 5 and 10 kHz are shown in figure 2b, c, respectively. At both frequencies, the effect of the bi-harmonic term expressing the influence of the cylinder’s thickness in equation (2.9) was found to be negligible. The destructive nature of the interfering wave in the shadow region becomes stronger owing to the flexibility of the cylinder. The finite length of the cylinder causes an interaction between the vertical wave modes, making equations (2.17) and (2.18) a matrix equation. While this interaction may increase the interfering wave behind the cylinder, calculations for infinite cylinders also

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reveal a more profound shadow region behind the flexible shell, indicating that the shell’s flexibility directly affects that region. Increasing the frequency to 10 kHz caused the shadow region to narrow down and also gave rise to some curious interference (according to the definition of Morse) patterns at the sidelines of the cylinder. Increasing the cylinder’s height from 8 to 12 m, i.e. 4 m above the water surface, had little effect on the results.

The external and internal sound fields present when the shell is filled with water are illustrated in figure 3. A validation exercise preceded these calculations in which the shell’s material was considered to be made of water, i.e. with a negligible Young’s modulus, resulting, as expected, in very small scattered and transmitted sound waves which were caused by the finite thickness of the shell; in which case, the difference between equations (2.17) and (2.18) becomes a finite difference approximation for the sound propagation. In the case of aluminium, the presence of water inside the shell reduced the interfering wave destruction effect behind the shell and made the scattered sound wave more similar to that of the rigid cylinder seen in figure 2a. A standing wave pattern is revealed inside the shell, where the wavefronts are bent owing to the circular geometry of the shell.

Filling the flexible shell with heavy liquid can reduce sound scattering but does not eliminate it. However, as previously shown, the equations actually allow a solution of complete scattering cancellation when an additional pressure, denoted as $f$, is applied at the shell walls. The distributions of the modulus $|f|$ are shown in figure 4 for the previously investigated frequencies of 5 and 10 kHz. While a very organized pattern is displayed for the lower frequency, which has a wavelength of the same magnitude as the shell’s diameter, a less organized pattern is shown for the higher frequency. Interestingly while both frequencies require maxima of $|f|$ around the line of impingement of the incident wave on the shell, i.e. at $\theta \approx 180^\circ$, the distribution of $|f|$ at the sidelines, i.e. $\theta \approx 90^\circ$ and $270^\circ$, is opposite between

Figure 3. Contours of the pressure modulus $|P|$ plotted (a) outside the cylindrical shell figure 2b but filled with water and (b) inside that shell. The sound frequency is 5 kHz and the rest of the conditions are the same as in figure 2.
the two frequencies. The latter result points to a connection with the difference seen at the sidelines of the scattered sound fields between the two frequencies, as illustrated in figure 2. Changing the vertical mode of the incident sound wave from $n = 0$ to $n = 1$ for the frequency of 5 kHz also resulted in an organized pattern for $|f|$. That pattern showed the same features for the variation in the $\theta$ direction, as seen in figure 4a for $n = 0$. However, it also yielded a minimum for $|f|$ in the vertical direction where the incident sound wave modulus was the minimum. Thus, the vertical variation of $|f|$ followed loosely the vertical variation of the modulus of the incident sound wave, as is also indicated by figure 4 for $n = 0$.

The distributions of the radial deflection $U$ of the shell for the frequency of 5 kHz with and without the applied pressure $f$ are shown in figure 5. The maxima of the radial deflection are at the incident wave impinging line $\theta \approx 180^\circ$, and its minima are at the sidelines, as in the distribution of $f$ that was required to yield zero sound scattering (figure 4a). Similar qualitative similarity between the two distributions of $U$ and $f$ was found for the higher frequency of 10 kHz. The distribution of $U$ when the scattered wave was cancelled is seen in figure 5b. This distribution only changes its amplitude as a function of frequency, as predicted by equation (2.8), and since only the incident sound pressure acts on the cylinder.

Numerical experimentation was carried out for discrete distributions of the applied pressure $f$ to minimize the amount of sound scattering by following the procedure outlined at the end of §2. A finite number of vertical or horizontal lines (rings) modelling the discrete distribution of $f$ was examined. Further constraints were placed on the distribution of $f$ along those lines either to follow the same vertical variation of the incident sound wave when vertical lines of $f$ were used or to be limited to the first few tangential modes for the horizontal lines distribution. These constraints were made in order to remove any possible dependency of the optimized distribution of $f$ on the grid resolution. Moderate success was achieved particularly with the vertical discrete distribution of $f$, leading to a reduction.
in the target function $Q$ defined in equation (2.19) by approximately 5 dB for 24 lines uniformly spaced in the tangential direction. The reduction in scattering was particularly noticeable in the interfering wave behind the cylinder although the reflected wave field was less affected, indicating the need for further work on this technique. Encouragingly, adding small random variations of up to 1 per cent to the spatial distribution of $f$ was found to cause very little change to $Q$, pointing to the existence of a flat rather than a sharp optimum point.

4. Summary

Free surface-piercing flexible, thin, cylindrical shells mounted vertically in shallow water were considered. Sound scattering by an external incident sound wave was investigated analytically and numerically by employing a linear theory of sound propagation and structural dynamics to account for the boundary conditions appropriate for shallow water and the effects of compressible fluid inside the shell. It was shown that the cylinder’s edge conditions caused coupling between the vertical-axis Fourier modes resulting in a matrix equation, which was solved after an orthogonalization procedure. A solution for zero sound scattering was shown to be possible under application of a continuous oscillating pressure distribution at the shell wall. Similarly, a solution for zero transmission of sound into the shell’s inner fluid was also found.

The proposed model was solved numerically for the case of a rigid aluminium shell, achieving excellent agreement with known results for a rigid cylinder. The effects of the shell’s flexibility were shown to enhance mainly the V-wake pattern behind the cylinder caused by the interfering wave. Filling the shell with a heavy fluid such as water reduced that effect, but some sound scattering nevertheless
remained. Contour distributions of the applied pressure needed for zero sound scattering were plotted, revealing organized structures for sound wavelengths of the same order as the shell’s diameter. The possibility of using a pre-determined discrete distribution of applied pressure was also examined, achieving moderate success in reducing the sound scattering, particularly in the interfering wave behind the cylinder.

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